Consensus reaching in committees and coalition formation

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EXTENDED ABSTRACT

The problem of consensus reaching appears very frequently. In particular, a committee has to solve this problem when choosing one or more alternatives from a set of applicants, proposals, etc. Using concepts introduced in [1], we formulate a consensus model for decision-making in committees. Decision makers may be advised to adjust their preferences in order to obtain a better consensus.

Let N be the set of all decision makers who try to reach consensus on some alternatives. Let A denote the set of all alternatives. Let C denote the set of all criteria. Each decision maker is supposed to have an evaluation of the importance of the criteria. Hence, for each $i \in N$, we assume $h_i : C \to [0,1]$, such that

$$\forall i \in N \ [\sum_{c \in C} h_i(c) = 1], \tag{1}$$

where $h_i(c)$ is i's evaluation (or weight) of criterion c. Moreover, for each $i \in N$, we also assume $g_i : C \times A \to [0,1]$ such that

$$\forall c \in C \ [\sum_{a \in A} g_i(c,a) = 1], \tag{2}$$

where $g_i(c,a)$ is the value of alternative a to decision maker i with respect to criterion c. Let $(h_i(c))_{c \in C}$ denote the $1 \times |C|$ matrix representing the evaluation (comparison) of the criteria by decision maker i, and let $(g_i(c,a))_{c \in C, a \in A}$ denote the $|C| \times |A|$ matrix containing i's evaluation (comparison) of all alternatives with respect to each criterion in C. For each $i \in N$, we define $f_i : A \to [0,1]$ such that

$$(f_i(a))_{a \in A} = (h_i(c))_{c \in C} \cdot (g_i(c,a))_{c \in C, a \in A}, \tag{3}$$

where $f_i(a)$ is i's evaluation of alternative a, and $(f_i(a))_{a \in A}$ is the $1 \times |A|$ matrix containing i's evaluation of each alternative.

The 'distances' between decision makers i and j are calculated as follows:

$$d(f_i, f_j) = \sqrt{\frac{1}{|A|} \sum_{a \in A} (f_i(a) - f_j(a))^2}. \tag{4}$$

By (1), (2), and (3), $0 \leq d(f_i, f_j) \leq 1$. A generalized consensus degree δ^* for a committee is defined as

$$\delta^* = 1 - d^* = 1 - \max\{d(f_i, f_j) \mid i, j \in N\}. \tag{5}$$

In the model a certain consensus degree $\widetilde{\delta}$ is required in advance. We say that a committee reaches consensus if $\delta^* \geq \widetilde{\delta}$.

We assume a kind of mediator, called the chairman. If the decision makers reach a consensus degree $\delta^* < \widetilde{\delta}$, it is the chairman who decides who should adjust his/her preferences in order to reach a better consensus. Let i^* denote a decision maker who will be advised to adjust his/her preferences. Let

$$D^* = \{i \in N \mid \exists j \in N \; [d(f_i, f_j) = d^*]\}. \tag{6}$$

$i^* \in D^*$ is a decision maker from D^* who satisfies the following condition

$$i^* = \arg\min_{k \in D^*} d^*_{-k}, \tag{7}$$

where d^*_{-k} is defined for $k \in D^*$ as

$$d^*_{-k} = \max\{d(f_i, f_j) \mid i, j \in N \setminus \{k\}\}. \tag{8}$$

If there are two members satisfying condition (7), the chairman chooses one of them. We assume a majority degree $\widetilde{m}$, that is, the minimal number of decision makers necessary to make a decision. The chairman's advice always leads to a increase of δ^*. If i^* refuses to change his/her preferences, he/she is 'excluded' from further discussion. If $|N \setminus \{i^*\}| \geq \widetilde{m}$, the remaining decision makers try to reach consensus. If $|N \setminus \{i^*\}| < \widetilde{m}$, the committee does not reach consensus. If i^* follows the chairman's advice, then the new generalized consensus degree δ'^* is calculated, and if $\delta'^* \geq \widetilde{\delta}$, the committee reaches consensus. If $\delta'^* < \widetilde{\delta}$, then a new decision maker, i'^*, is appointed by the chairman for adjusting his/her preferences, etc.

If consensus is reached by the committee, that is, if the generalized (final) consensus degree is not smaller than $\widetilde{\delta}$, a mean consensus decision is calculated. Let $N^* \subseteq N$ denote the set of the decision makers who succeeded in reaching consensus. Assuming that the decision makers might be unequally 'important', we add up the weighted (final) values of the alternatives to all decision makers from N^*. For each $a \in A$, the weighted value $f(a)$ of alternative a is defined as

$$f(a) = \sum_{i \in N^*} w'_i \cdot f_i(a), \tag{9}$$

where for each $i \in N^*$

$$w'_i = \frac{w_i}{\sum_{j \in N^*} w_j}, \tag{10}$$

and w_i means the 'weight' of decision maker $i \in N^*$. The committee chooses the alternatives with the greatest value of $f(a)$. In particular, if only one alternative may be chosen, the committee chooses the alternative a^* such that

$$a^* = \arg\max_{a \in A} f(a). \tag{11}$$

If there are (at least) two alternatives satisfying condition (11), and only one alternative may be chosen, the chairman decides for one of them.

The consensus model may be also applied to a model of coalition formation ([2]). If parties are willing to compromise, it is always possible to reach consensus, and to create a feasible government. In the procedure there is also an 'outsider', called the chairman, who advises parties how to adjust their preferences. First, each feasible coalition tries to reach consensus within this coalition about the government to be formed. Parties consider only feasible governments, i.e., governments acceptable for all parties belonging to the coalition involved, and if there is only one feasible government they can form, they agree. If the parties from a given coalition manage to reach consensus, the coalition proposes to form the government agreed upon. This consensus government is stable in the given coalition with respect to the set of all feasible governments formed by that coalition. It may happen, of course, that no feasible coalition reaches consensus. In this case, no final government is created. If there is only one feasible coalition which reaches consensus, then the government proposed by this coalition is formed. If there are at least two coalitions that succeed in reaching consensus, that is, if at least two governments are proposed, we select the governments which are 'internally stable'. Next, if there are at least two such governments, an extra procedure is applied in order to choose one of these governments. We construct several such procedures. In the paper, simple examples are presented.

References

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